Philadelphia University **Student Name:**

Faculty of Engineering **Student Number:**

Dept. of Mechatronics Engineering

Mid Exam, First Semester: 2013/2014

Course Title: Advanced Engineering Mathematics	**Date: 5/12/2013**
Course No: (640711) – Msc. Course	**Time Allowed: 1.5 Hour**
Lecturer: Dr. Mohammed Mahdi	**No. of Pages: 2**

Question 1: (20 Marks)

Objectives: This question is about the basic concepts of Laplace and z transforms.

Evaluate the following using Laplace and z transforms formulas: -

- Laplace transform of $t^3 e^{-3t}$ (5 Marks)
- Laplace inverse of $\frac{1}{s\ (s^2+1)}$ (5 Marks)
- Prove that Laplace inverse of F(s-a) is $e^{at} f(t)$ (5 Marks)
- Prove that z{t} = $\frac{Tz}{(z-1)^2}$ (5 Marks)

Question 2: (40 Marks)

Objectives: This question is about Laplace inverse and solving differential equations using Laplace transform.

A) For $F(s)=\frac{(2s-7)}{(s^2+25)}$, it requires to find: -

- **Its related controllable canonical state space representation matrices**. (5 Marks)
- **Its related diagonal state space representation matrices.** (10 marks)
- **f (t) using Laplace inverse properties.** (5 Marks)

B) Solve $\ddot{y}+9y=18t$ with y (0) =0, $\dot{y}$ (0) =6. (10 Marks)

C) Given $\ddot{y}-6y=g(t)$, with $y(0)=\dot{y}(0)=0$, and g (t) = $\begin{cases} 0 & t \prec \pi \\ \sin(t-\pi) & t \geq \pi \end{cases}$, **it is required to show y(s) in factorized form.** (10 Marks)

Question 3: (40 Marks)

Objectives: This question is about the solution of state space representation mathematical model.

A) Given $\ddot{y}+2\dot{y}+y=x$. It is required to find the matrix exponential (e^{At}) **using Cayley-Hamilton theorem.** (10 Marks)

B) For a system with A=$\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$, B=$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, C=$\begin{bmatrix} 0 & 1 \end{bmatrix}$, and D=0. The matrix exponential (e^{At}) is found to be $\begin{bmatrix} 3e^{-2t}-2e^{-3t} & e^{-2t}-e^{-3t} \\ -6e^{-2t}+6e^{-3t} & -2e^{-2t}+3e^{-3t} \end{bmatrix}$. It is required to find:-

- **System transfer function.** (10 Marks)
- **Initial and final values for unit impulse change in input**. **Discuss your results.** (10 Marks)
- **Output response y (t) to a unit impulse with zero initial conditions using the given state space representation.** (10 Marks)

Good Luck